Automatic Control

Chapter five

Stability analysis

By

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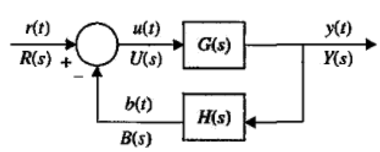

- The transfer function M(s) can be written as:

$$M(s)=\frac{Y(s)}{R(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a^n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

- The function M(s) may be:
  1. Strictly proper if n>m
  2. proper if n=m
  3. improper if n<m

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Characteristic equation

- The dominator of the transfer function M(s)equal to zero is called the characteristic equation.

$$a^n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

- The zeros of this equation are called the **poles** of the transfer function
- The zeros of the characteristic equation have an important role in the stability of the system.
- Stability means that the output dose not converge to the desired output properly (i.e. oscillate over the output) or absolutely diverge from the desired output

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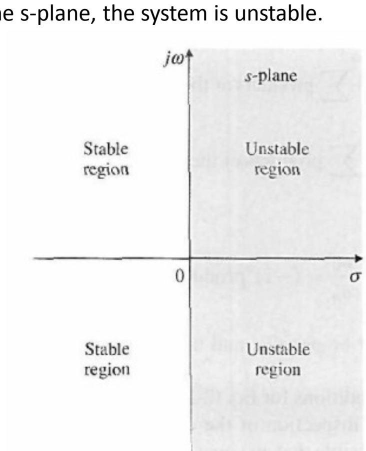
Stability conditions

- It was found that if at least one of the roots of the characteristic equation lies at the right side of the s-plane, the system is unstable.



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Stability conditions

❑A further analysis is shown in the following table

TABLE 2-5 Stability Conditions of Linear Continuous-Data Time-Invariant SISO Systems

Stability Condition	Root Values
Asymptotically stable or simply stable	$\sigma_i < 0$ for all i, $i = 1, 2, \ldots, n$. (All the roots are in the left-half s-plane.)
Marginally stable or marginally unstable	$\sigma_i = 0$ for any i for simple roots, and no $\sigma_i > 0$ For $i = 1, 2, \ldots, n$ (at least one simple root, no multiple-order roots on the $j\omega$-axis, and n roots in the right-half s-plane; note exceptions)
Unstable	$\sigma_i > 0$ for any i, or $\sigma_i = 0$ for any multiple-order root; $i = 1, 2, \ldots, n$ (at least one simple root in the right-half s-plane or at least one multiple-order root on the $j\omega$-axis)

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Routh’s method

❑It is not necessary to find the roots of the characteristic equation to decide if the system is stable or not.

❑Routh give us a method to find if the system is stable or not without solving

❑The roots of the equation are all in the left half of the .s-plane if all the elements of the first column of the Routh's tabulation are of the same sign. The number of changes of signs in the elements of the first column equals the number of roots with positive real parts, or those in the right-half .v-plane.

Stability analysis

A u t o m a t i c C o n t r o l

Routh's method

- Example

$$a_6 s^6 + a_5 s^5 + \cdots + a_1 s + a_0 = 0 \qquad (2\text{-}252)$$

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$\frac{a_5 a_4 - a_6 a_3}{a_5} = A$	$\frac{a_5 a_2 - a_6 a_1}{a_5} = B$	$\frac{a_5 a_0 - a_6 \times 0}{a_5} = a_0$	0
s^3	$\frac{A a_3 - a_5 B}{A} = C$	$\frac{A a_1 - a_5 a_0}{A} = D$	$\frac{A \times 0 - a_5 \times 0}{A} = 0$	0
s^2	$\frac{BC - AD}{C} = E$	$\frac{C a_0 - A \times 0}{C} = a_0$	$\frac{C \times 0 - A \times 0}{C} = 0$	0
s^1	$\frac{ED - C a_0}{E} = F$	0	0	0
s^0	$\frac{F a_0 - E \times 0}{F} = a_0$	0	0	0

Stability analysis

A u t o m a t i c C o n t r o l

Routh's method

- Example $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$

	s^4	2	3	10
	s^3	1	5	0
Sign change				
	s^2	$\frac{(1)(3) - (2)(5)}{1} = -7$	10	0
Sign change				
	s^1	$\frac{(-7)(5) - (1)(10)}{-7} = 6.43$	0	0
	s^0	10	0	0